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Hard diffractive photon-proton scattering at large $|t|$

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Abstract. We propose to test perturbative QCD(pQCD) in the Regge limit by means of diffractive photon scattering, $\gamma p \to \gamma X$, at large $|t|$ and very high energies, $W^2 \gg |t| \gg \Lambda^2_{QCD}$. The helicity amplitudes of this process were calculated using the Lipatov solution of the BFKL equation for $t \neq 0$. We found that the perturbatively calculated cross section for this process is comparable in magnitude to the cross section for J/Ψ photoproduction assuming similar kinematics.

1 Introduction

In the quest for more and better tests of pQCD in the Regge limit, better known as BFKL [1], we suggest hard diffractive photon-proton scattering at large momentum transfer $|t|$. This process is closely related to the diffractive production of vector mesons at large $|t|$ [2–4], where the vector meson in the final state takes the place of the photon. Although suppressed by an extra α_{em}, diffractive γp-scattering has the great advantage of being completely calculable. No phenomenological input in terms of a vector meson wave function is needed. The signature of this process is also very clear. The photoproduced photon is scattered into the backward region of the detector at a very low angle. The transverse momentum transferred from the photon is balanced by a jet in the forward region, which does not need to be resolved. More immportant is the very large rapidity gap between the photon in the backward and the hadronic system in the forward region. The idea to use this process as a test for pQCD was already discussed in [5].

A very precise definition of the cross section is not necessary, since we are interested in a rough estimate. Indeed, since BFKL enters on the level of the amplitude the resulting enhancement is very large and therefore the theoretical uncertainty as well. We work with the leading order BFKL-solution which is by now known to receive strong NLO-corrections [6]. Already a reduction by factor of 1/2 in the LO-BFKL-kernel leads to an order of magnitude reduction in the elastic cross section. Another source of uncertainty is the influence of nonperturbative effects despite the fact that we require a large momentum transfer $|t|$. On the proton side it has been proven [7,8] that the large $|t|$-Pomeron-exchange factorizes in the sense that it directly couples to partons. The difference in the coupling to quarks and gluons is only a trivial colour factor, and the corresponding parton distribution can be taken from any conventional LO-pdf. As we already pointed out we are at the present only interested in a first rough estimate of the pQCD predictions and thus focus on the elastic photon-quark scattering. For comparison we include in our numerical analysis the diffractive production of J/Ψ and find that both cross sections are close in magnitude. We also perform a Vector meson Dominance Model (VDM) inspired estimate for the same cross sections, which shows that with BFKL the γq-cross section for large $|t|$ can hardly be matched with the VDM-result at low $|t|$. We hope, of course, that the γq-cross section will be measured at HERA in the near future.

In the technical part of this paper we extend the successful concept of the photon wave function [12,13] to include nonzero momentum transfer. We then convolute the corresponding expression for the photon wave function directly in impact-parameter space with the conformal eigenfunction of the nonforward BFKL-solution [10]. This part is presented in a rather detailed way to illustrate some of the techniques which might be useful in other related cases, for it seems most appropriate in dealing with integrals of two-dimensional, conformal field theories. The impact factor describing the coupling of two gluons via a quark-box to the photons has effectively been already calculated in QED in the context of photon-photon transition [9,11], only colour factors needed to be added in the case of QCD. Since for our calculation we need a somewhat different form of the impact factor than found in the literature, we have reconsidered this calculation using slightly different methods.

The paper contains three technical sections that deal with the derivation of the γq-cross section. The following section is devoted to the generalization of the photon wave function to include momentum transfer, in Sect. 3 we explain the convolution of the wave function with the conformal BFKL-eigenfunctions, and in Sect. 4 we collect all pieces to derive the final expression for the cross section.

In Sect. 5 we present the numerical results and conclude with Sect. 6.

2 Photon wave function

In the standard case of deep inelastic scattering the photon appears only in the initial state, and its wave function [12,13] can be formulated in the simplifying photon-proton CMS. In this frame the momentum of the photon has only longitudinal components and no transverse. When we consider, however, photon elastic scattering we have two photons, one in the initial state and the second in the final state of the process. We can still choose a frame in which one of the photons has only longitudinal components, but the second photon receives transverse components due to the momentum transfer q. We therefore have to generalize the photon wave function description to also include transverse components. We use the standard notation for deep inelastic scattering such as Q' (final state photon) and p (proton). The light cone vectors Q' and p define the CMS we work in. For our discussion it is easier to assume that the outgoing photon, which includes the momentum transfer q, lies along the z-axis of our system and the incoming photon with the momentum Q has components in the transverse plane, i.e. $Q = Q' + q$. The corresponding polarization vector for the incoming photon reads:

$$\epsilon(\pm)_\mu \;=\; \epsilon(\pm)_{\perp\mu} \;-\; \frac{q_\perp \cdot \epsilon(\pm)_\perp}{p \cdot Q}\, p_\mu \qquad (2.1)$$

with two helicity states ($\pm$) in the transverse plane of our frame:

$$\epsilon_\perp(\pm) \;=\; \frac{1}{\sqrt{2}}(0,1,\pm i,0)\;. \qquad (2.2)$$

The photon couples to a quark-antiquark pair with the momenta k_1 and k_2, respectively. Using Sudakov decomposition we write the momenta as ($s = 2Q' \cdot p = 4Q'_0 p_0$):

$$\begin{aligned} q &= \beta_q\, p + q_\perp\;, \\ k_1 &= \alpha\, Q' + \left(\beta_q + \frac{k^2_{2_\perp}}{(1-\alpha)\, s}\right) p + k_{1_\perp}\;, \qquad (2.3) \\ k_2 &= (1-\alpha)\, Q' - \frac{k^2_{2_\perp}}{(1-\alpha)\, s}\, p + k_{2_\perp}\;. \end{aligned}$$

One of these quark momenta is offshell depending on where the t-channel gluon couples. In the case discussed here it is k_1 as indicated in Fig. 1. The momentum k_1 can be made onshell by adding a component with respect to p. We make use of this trick in order to break up the trace of the quark loop by inserting quark spinors with onshell momenta. This can be done without effecting the final result, because the adjacent gluon coupling has as leading component a $\not p$, which cancels our proposed modification in the trace. In the trace we therefore substitute k_1 by $\tilde{k}_1$:

$$\tilde{k}_1 \;=\; \alpha\, Q' - \frac{k^2_{1_\perp}}{\alpha\, s}\, p + k_{1_\perp}\;. \qquad (2.4)$$

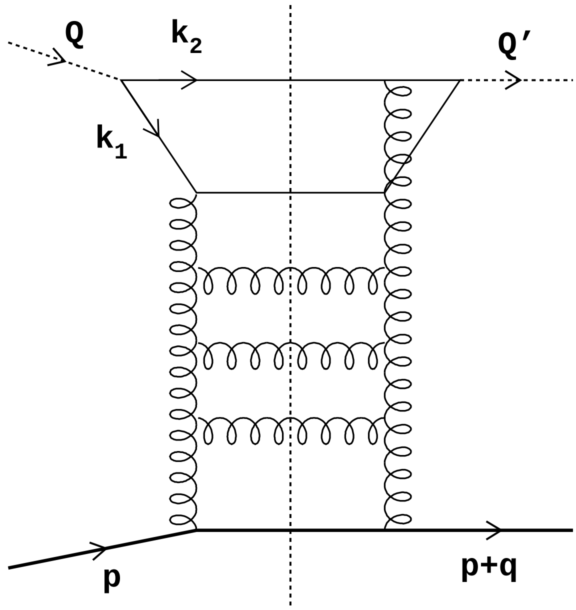

Fig. 1. Diagrammatical representation of elastic γq-scattering with BFKL-ladder exchange

The denominator of our wave function contains of course the original virtuality k_1^2. In the following, we will change the transverse components for the momenta from Minkowski space to Euclidean, i.e. $k_\perp \to \mathbf{k}$ with $k_\perp^2 = -\mathbf{k}^2$, and despite the fact that the photon virtuality is negative we will use the convention that $Q^2 = |Q^2| = -\beta_q s + \mathbf{q}^2$.

After these preliminaries it is rather straightforward to introduce the photon wave function ($h = \pm$ for the quark helicity, $h_\gamma = \pm$ for the photon helicity):

$$\begin{aligned} \Psi_{(h_\gamma,h)}(\mathbf{k}_1,\mathbf{k}_2,\alpha) &= e\, \frac{\bar{u}(k_2,h)\, \not\epsilon(h_\gamma)\, u(\tilde{k}_1,h)\sqrt{\alpha(1-\alpha)}}{[\mathbf{k}_1-\alpha\mathbf{q}]^2+\alpha(1-\alpha)Q^2} \\ &\quad \times (2\pi)^2\delta^2(\mathbf{q}-\mathbf{k}_1-\mathbf{k}_2)\;. \qquad (2.5) \end{aligned}$$

In the expression above we have made use of the relation $v(\tilde{k}_1,-h) = u(\tilde{k}_1,h)$, and we have already incorporated a factor α from the gluon vertex and a phase space contribution $1/\sqrt{\alpha(1-\alpha)}$. The propagator itself has the form $|k_1^2| = ([\mathbf{k}_1-\alpha\mathbf{q}]^2+\alpha(1-\alpha)Q^2)/(1-\alpha)$. In the chiral representation the spinors are of the following form (in complex notation for $\mathbf{k}_1$):

$$\begin{aligned} u(\tilde{k}_1,+) &= \left(\frac{\mathbf{k}_1^*}{\sqrt{2\alpha Q'_0}}\;,\;\sqrt{2\alpha Q'_0}\;,\;0\;,\;0\right)\;, \qquad (2.6) \\ u(\tilde{k}_1,-) &= \left(0\;,\;0,\;\sqrt{2\alpha Q'_0}\;,\;\frac{-\mathbf{k}_1}{\sqrt{2\alpha Q'_0}}\right)\;, \end{aligned}$$

and similar expressions for $u(k_2,h)$ by substituting $\mathbf{k}_1$ by $\mathbf{k}_2$ and α by $1-\alpha$.

Equation (2.5) together with (2.1) and (2.6) yields the following expressions for the wave function (for better readability we present all helicity states explicitly):

$$\Psi_{(+,+)}(\mathbf{k}_1,\mathbf{k}_2,\alpha) = -\frac{\sqrt{2}\, e\, \alpha\, [\mathbf{k}_2-(1-\alpha)\mathbf{q}]}{[\mathbf{k}_2-(1-\alpha)\mathbf{q}]^2+\alpha(1-\alpha)Q^2} \times(2\pi)^2\delta^2(\mathbf{q}-\mathbf{k}_1-\mathbf{k}_2)\,,$$
$$\Psi_{(+,-)}(\mathbf{k}_1,\mathbf{k}_2,\alpha) = -\frac{\sqrt{2}\, e\, (1-\alpha)\, [\mathbf{k}_1-\alpha\mathbf{q}]}{[\mathbf{k}_1-\alpha\mathbf{q}]^2+\alpha(1-\alpha)Q^2} \times(2\pi)^2\delta^2(\mathbf{q}-\mathbf{k}_1-\mathbf{k}_2)\,,$$
$$\Psi_{(-,+)}(\mathbf{k}_1,\mathbf{k}_2,\alpha) = -\frac{\sqrt{2}\, e\, (1-\alpha)\, [\mathbf{k}_1^*-\alpha\mathbf{q}^*]}{[\mathbf{k}_1-\alpha\mathbf{q}]^2+\alpha(1-\alpha)Q^2} \times(2\pi)^2\delta^2(\mathbf{q}-\mathbf{k}_1-\mathbf{k}_2)\,, \tag{2.7}$$
$$\Psi_{(-,-)}(\mathbf{k}_1,\mathbf{k}_2,\alpha) = -\frac{\sqrt{2}\, e\, \alpha\, [\mathbf{k}_2^*-(1-\alpha)\mathbf{q}^*]}{[\mathbf{k}_2-(1-\alpha)\mathbf{q}]^2+\alpha(1-\alpha)Q^2} \times(2\pi)^2\delta^2(\mathbf{q}-\mathbf{k}_1-\mathbf{k}_2)\,.$$

One could of course apply momentum conservation $\mathbf{k}_2 = \mathbf{q}-\mathbf{k}_1$ and rewrite all equations in terms of $\mathbf{k}_1$ only. But it is the form presented in (2.8) which emerges first and still exhibits the fact that the same factor $\alpha(1-\alpha)$ appears in front of $\mathbf{q}$ in the numerator of all expressions. The latter is a result of the longitudinal contribution p_μ in the polarization vector (2.1), which is the same for all quark helicities.

We transform into impact-parameter space by taking the Fourier transform with respect to $\mathbf{k}_1$ and $\mathbf{k}_2$:

$$\int \frac{d^2\mathbf{k}_1 d^2\mathbf{k}_2}{(2\pi)^4}\, \Psi(\mathbf{k}_1,\mathbf{k}_2,\alpha)\, \mathrm{e}^{i\mathbf{k}_1\cdot\mathbf{r}_1}\, \mathrm{e}^{i\mathbf{k}_2\cdot\mathbf{r}_2} = \Psi(\mathbf{r}_1,\mathbf{r}_2,\alpha)\,, \tag{2.8}$$

and arrive at:

$$\Psi_{(+,+)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, \alpha\, \sqrt{\alpha(1-\alpha)Q^2} \times K_1(\sqrt{\alpha(1-\alpha)Q^2}\, |\mathbf{r}|)\, \frac{\mathbf{r}}{|\mathbf{r}|}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\, \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\,,$$
$$\Psi_{(+,-)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, (1-\alpha)\, \sqrt{\alpha(1-\alpha)Q^2} \times K_1(\sqrt{\alpha(1-\alpha)Q^2}\, |\mathbf{r}|)\, \frac{\mathbf{r}}{|\mathbf{r}|}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\, \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\,,$$
$$\Psi_{(-,+)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, (1-\alpha)\, \sqrt{\alpha(1-\alpha)Q^2} \times K_1(\sqrt{\alpha(1-\alpha)Q^2}\, |\mathbf{r}|)\, \frac{\mathbf{r}^*}{|\mathbf{r}|}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\, \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\,,$$
$$\Psi_{(-,-)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, \alpha\, \sqrt{\alpha(1-\alpha)Q^2} \times K_1(\sqrt{\alpha(1-\alpha)Q^2}\, |\mathbf{r}|)\, \frac{\mathbf{r}^*}{|\mathbf{r}|}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\, \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2} \tag{2.9}$$

with $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and K_1 being the MacDonald (Bessel) function. The whole $\mathbf{q}$-dependence is shifted into phase factors, and the rest is the same as for the 'forward' case, i.e. $\mathbf{q} = 0$.

One could now go ahead and work with the above wave function for virtual photons, i.e study the scattering of virtual photons into virtual photons. At this point we would like to keep the analysis a bit simpler and consider only real photons. To this end we take $Q^2 \to 0$ and (2.9) reduces to:

$$\Psi_{(+,+)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, \alpha\, \frac{\mathbf{r}}{|\mathbf{r}|^2}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\, \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\,,$$
$$\Psi_{(+,-)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, (1-\alpha)\, \frac{\mathbf{r}}{|\mathbf{r}|^2}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1} \times\mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\,,$$
$$\Psi_{(-,+)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, (1-\alpha)\, \frac{\mathbf{r}^*}{|\mathbf{r}|^2}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1} \times\mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\,, \tag{2.10}$$
$$\Psi_{(-,-)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = \frac{\sqrt{2}\, i\, e}{2\pi}\, \alpha\, \frac{\mathbf{r}^*}{|\mathbf{r}|^2}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\, \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\,.$$

In the next step we have to consider the wave function for the outgoing photon Ψ^*. It is basically the complex conjugate of the previous expression except for the phase factors, which are absent because the momentum for the outgoing photon is simply Q', i.e. $q = 0$:

$$\Psi^*_{(+,+)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = -\frac{\sqrt{2}\, i\, e}{2\pi}\, \alpha\, \frac{\mathbf{r}^*}{|\mathbf{r}|^2}\,,$$
$$\Psi^*_{(+,-)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = -\frac{\sqrt{2}\, i\, e}{2\pi}\, (1-\alpha)\, \frac{\mathbf{r}^*}{|\mathbf{r}|^2}\,,$$
$$\Psi^*_{(-,+)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = -\frac{\sqrt{2}\, i\, e}{2\pi}\, (1-\alpha)\, \frac{\mathbf{r}}{|\mathbf{r}|^2}\,, \tag{2.11}$$
$$\Psi^*_{(-,-)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) = -\frac{\sqrt{2}\, i\, e}{2\pi}\, \alpha\, \frac{\mathbf{r}}{|\mathbf{r}|^2}\,.$$

The impact factor is essentially the product of the two sets of wave functions, since the imaginary part of our amplitude dominates and all intermediate quarks are onshell. With regard to the two t-channel gluons located at ρ_1 and ρ_2, we have to make sure that each of the gluons couples to each of the quarks (g is the strong coupling constant):

$$\Phi_{(h_\gamma,h_\gamma^*)}(\rho_1,\rho_2) = g^2 \sum_{h=\pm} \int_0^1 d\alpha \int d^2\mathbf{r}_1 d^2\mathbf{r}_2\, \Psi_{(h_\gamma,h)} \times(\mathbf{r}_1,\mathbf{r}_2,\alpha)\, \Psi^*_{(h_\gamma^*,h)}(\mathbf{r}_1,\mathbf{r}_2,\alpha) \times[\delta^2(\mathbf{r}_1-\rho_1)-\delta^2(\mathbf{r}_2-\rho_1)] \times[\delta^2(\mathbf{r}_1-\rho_2)-\delta^2(\mathbf{r}_2-\rho_2)]\,. \tag{2.12}$$

The normalization will be adjusted later in the full expression for the amplitude. Spelling out the previous expression we find:

$$\Phi_{(+,+)}(\rho_1,\rho_2) = \tag{2.13}$$
$$\Phi_{(-,-)}(\rho_1,\rho_2) = \frac{e^2 g^2}{2\pi^2} \int_0^1 d\alpha \int d^2\mathbf{r}_1 d^2\mathbf{r}_2\, [\alpha^2+(1-\alpha)^2] \times\frac{1}{|\mathbf{r}|^2}\, \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\, \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2} \times[\delta^2(\mathbf{r}_1-\rho_1)-\delta^2(\mathbf{r}_2-\rho_1)]$$

$$\times[\delta^2(\mathbf{r}_1-\rho_2)-\delta^2(\mathbf{r}_2-\rho_2)]\ ,$$

$$\begin{aligned}\Phi_{(+,-)}(\rho_1,\rho_2) = {}& \frac{e^2g^2}{\pi^2}\int_0^1 d\alpha \int d^2\mathbf{r}_1 d^2\mathbf{r}_2\ \alpha(1-\alpha)\\ &\times\frac{\mathbf{r}^2}{|\mathbf{r}|^4}\ \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\ \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\\ &\times[\delta^2(\mathbf{r}_1-\rho_1)-\delta^2(\mathbf{r}_2-\rho_1)]\\ &\times[\delta^2(\mathbf{r}_1-\rho_2)-\delta^2(\mathbf{r}_2-\rho_2)]\ ,\end{aligned}$$

$$\begin{aligned}\Phi_{(-,+)}(\rho_1,\rho_2) = {}& \frac{e^2g^2}{\pi^2}\int_0^1 d\alpha \int d^2\mathbf{r}_1 d^2\mathbf{r}_2\ \alpha(1-\alpha)\\ &\times\frac{\mathbf{r}^{*2}}{|\mathbf{r}|^4}\ \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\ \mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\\ &\times[\delta^2(\mathbf{r}_1-\rho_1)-\delta^2(\mathbf{r}_2-\rho_1)]\\ &\times[\delta^2(\mathbf{r}_1-\rho_2)-\delta^2(\mathbf{r}_2-\rho_2)]\ .\end{aligned}$$

$\Phi_{(+,+)}$ and $\Phi_{(-,-)}$ are contributions without helicity flip and $\Phi_{(+,-)}$ and $\Phi_{(-,+)}$ are those with flip. It is not apparent from (2.13) that the two helicity flip contributions are the same, but the following calculation will show that they coincide as one may expect on general grounds. In the end it is enough to deal with a single helicity flip and a single helicity non-flip amplitude.

3 Projection on conformal eigenstates

In this section we exploit the properties of conformal invariance of the BFKL-equation [10], which can be solved in terms of the conformal covariant eigenfunctions:

$$E^{\nu}(\rho_{10},\rho_{20}) \ = \ \left|\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right|^{1+2i\nu}\ , \qquad (3.1)$$

where ν is the conformal weight. We have ignored the conformal spin n that is required to form a complete set. In practice, though, any contribution with $n\neq 0$ gives a subleading contribution at high energies. If one wished to do so, the following calculation can be generalized to include n.

In projecting the impact factor on the eigenfunction we have to perform the following integration ($\rho_{10}=\rho_1-\rho_0$, etc.):

$$\int d^2\rho_1 d^2\rho_2\ \Phi_{(h_\gamma,h^*_\gamma)}(\rho_1,\rho_2)\ E^{\nu}(\rho_{10},\rho_{20})\ . \qquad (3.2)$$

We realize that in two terms of (2.13) the δ-function forces ρ_1 and ρ_2 into one point which results in a vanishing contribution due to a zero in E^ν ($Re[1+2i\nu]$ has to be kept positive). The second observation is the symmetry in ρ_1 and ρ_2 which allows one to write the whole amplitude in one term multiplied by 2 (non-flip here):

$$\begin{aligned}&\frac{e^2g^2}{\pi^2}\int_0^1 d\alpha\ [\alpha^2+(1-\alpha)^2]\int d^2\mathbf{r}_1 d^2\mathbf{r}_2\ \frac{1}{|\mathbf{r}|^2}\ \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}_1}\\ &\quad\times\mathrm{e}^{i(1-\alpha)\mathbf{q}\cdot\mathbf{r}_2}\ \left|\frac{\mathbf{r}}{(\mathbf{r}_1-\rho_0)(\mathbf{r}_2-\rho_0)}\right|^{1+2i\nu}\ . \qquad (3.3)\end{aligned}$$

We will shift $\mathbf{r}_1$ and $\mathbf{r}_2$ by ρ_0 generating an overall phase factor and then substitute $\mathbf{r}_1$ by $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$:

$$\begin{aligned}&\frac{e^2g^2}{\pi^2}\ \mathrm{e}^{i\mathbf{q}\cdot\rho_0}\int_0^1 d\alpha\ [\alpha^2+(1-\alpha)^2]\int d^2\mathbf{r}\ d^2\mathbf{r}_2\ \mathrm{e}^{i\alpha\mathbf{q}\cdot\mathbf{r}}\\ &\quad\times\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}_2}\ |\mathbf{r}|^{-1+2i\nu}\ |(\mathbf{r}+\mathbf{r}_2)\ \mathbf{r}_2|^{-1-2i\nu}\ . \qquad (3.4)\end{aligned}$$

The overall phase factor will be ignored for the moment, and will be reconsidered at the end of this section. We carry on in our calculation shifting $\mathbf{r}_2$ by $-\alpha\ \mathbf{r}$, eliminating the phase factor that depends on $\mathbf{r}$:

$$\begin{aligned}&\frac{e^2g^2}{\pi^2}\int_0^1 d\alpha\ [\alpha^2+(1-\alpha)^2]\int d^2\mathbf{r}\ d^2\mathbf{r}_2\ \ \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}_2}\ |\mathbf{r}|^{-1+2i\nu}\ |\\ &\quad\times[\mathbf{r}_2+(1-\alpha)\mathbf{r}]\ [\mathbf{r}_2-\alpha\mathbf{r}]|^{-1-2i\nu}\ . \qquad (3.5)\end{aligned}$$

We then switch from the conventional representation of the transverse vectors to the corresponding complex notation, i.e from $\mathbf{r}=(r_1,r_2)$ to $a=r_1+i\ r_2$ and $b=r_1-i\ r_2$, and make use of the freedom to rotate our system such that $\mathbf{q}$ is real:

$$\begin{aligned}&-\frac{e^2g^2}{4\pi^2}\int_0^1 d\alpha\ [\alpha^2+(1-\alpha)^2]\\ &\times\int da\ db\ da_2\ db_2\ \ \mathrm{e}^{iq/2(a_2+b_2)}\ (a\ b)^{-1/2+i\nu}\\ &\times\{[a_2+(1-\alpha)a]\ [b_2+(1-\alpha)b]\\ &\times[a_2-\alpha\ a][b_2-\alpha\ b]\}^{-1/2-i\nu}\ . \qquad (3.6)\end{aligned}$$

In what follows we rescale a and b by a_2 and b_2, respectively, and thus factorize the integration in a,b and a_2,b_2:

$$\begin{aligned}&-\frac{e^2g^2}{4\pi^2}\int_0^1 d\alpha\ [\alpha^2+(1-\alpha)^2]\\ &\times\int da_2\ db_2\ \ \mathrm{e}^{iq/2(a_2+b_2)}\ (a_2\ b_2)^{-1/2-i\nu}\\ &\times\int da\ db\ \ (a\ b)^{-1/2+i\nu}\ \{[1+(1-\alpha)a]\ [1+(1-\alpha)b]\\ &\times[1-\alpha\ a][1-\alpha\ b]\}^{-1/2-i\nu}\ . \qquad (3.7)\end{aligned}$$

We now rescale a_2 and b_2 by $2i/q$:

$$\begin{aligned}&\frac{e^2g^2}{4\pi^2}\ \left(\frac{2}{q}\right)^{1-2i\nu}\int_0^1 d\alpha\ [\alpha^2+(1-\alpha)^2]\\ &\times\int da_2\ db_2\ \ \mathrm{e}^{-(a_2+b_2)}\ (-a_2\ b_2)^{-1/2-i\nu}\\ &\times\int da\ db\ \ (a\ b)^{-1/2+i\nu}\ \{[1+(1-\alpha)a]\ [1+(1-\alpha)b]\\ &\times[1-\alpha\ a][1-\alpha\ b]\}^{-1/2-i\nu}\ , \qquad (3.8)\end{aligned}$$

and note the minus sign in the factor $(-a_2\ b_2)^{-1/2-i\nu}$.

At this point we have to consider the analytic structure of our integrand and the proper path of integration. We focus on the a_2-integration by keeping b_2 fixed, quasi as a parameter. With the minus sign there is a cut in the complex a_2 plane to the right when $b_2>0$ or to the left when $b_2 < 0$. Since the exponent also has a minus sign, we would like to shift the integration contour to the right. A nonzero contribution only arises if a singularity is encountered, i.e. if b_2 is positive. Closing the contour around the cut means we have to take the discontinuity along the cut and then integrate in a_2 over the positive axis:

$$\begin{aligned}
&\int da_2\, db_2\, \mathrm{e}^{-(a_2+b_2)}\, (-a_2\, b_2)^{-1/2-i\nu} \\
&= -\,2i\, \sin[(-1/2-i\nu)\pi] \\
&\quad\times \int_0^\infty da_2\, a_2^{-1/2-i\nu}\, \mathrm{e}^{-a_2} \int_0^\infty db_2\, b_2^{-1/2-i\nu}\, \mathrm{e}^{-b_2}\ , \\
&= -\,2i\, \sin[(-1/2-i\nu)\pi]\, \Gamma^2(1/2-i\nu)\ . \qquad (3.9)
\end{aligned}$$

The $-\,2i\ \sin[(-1/2-i\nu)\pi]$ is the result of taking the discontinuity.

In a similar way the integration over a and b can be performed. We again keep b fixed and consider the integration over a. The integrand is convergent even without the presence of an exponent. There will be a nonzero contribution only if the integration contour runs between two cuts, those two cuts which emerge on both sides of the complex a-plane when $-1/(1-\alpha) < b < 1/\alpha$. We have to contend, however, with another cut due to the factor $(a\ b)^{-1/2+i\nu}$ in (3.8). This lies to the right or to the left depending on the sign of b. Therefore, we have to consider two cases: $-1/(1-\alpha) < b < 0$ and $0 < b < 1/\alpha$, closing the contour to that side which has only one cut:

$$\begin{aligned}
&\int da\ db\ \ (a\ b)^{-1/2+i\nu}\ \{[1+(1-\alpha)a]\ [1+(1-\alpha)b] \\
&\quad\times[1-\alpha\ a][1-\alpha\ b]\}^{-1/2-i\nu} \\
&= 2i\ \sin[(-1/2-i\nu)\pi]\ \left\{\int_{-1/(1-\alpha)}^{0} db\ \int_{-\infty}^{-1/(1-\alpha)} da \right. \\
&\quad\left. + \int_0^{1/\alpha} db\ \int_{1/\alpha}^{\infty} da\right\}(a\ b)^{-1/2+i\nu} \\
&\quad\times\{-[1+(1-\alpha)a]\ [1+(1-\alpha)b] \\
&\quad\times[1-\alpha\ a][1-\alpha\ b]\}^{-1/2-i\nu}\ , \qquad (3.10) \\
&= 4i\ \ \sin[(-1/2-i\nu)\pi]\ [\alpha(1-\alpha)]^{-1/2-i\nu} \\
&\quad\times \int_0^1 db\, b^{-1/2+i\nu}\ \left\{[1-b][1+\frac{\alpha}{1-\alpha}b]\right\}^{-1/2-i\nu} \\
&\quad\times \int_0^1 da\ a^{-1/2+i\nu}\ \left\{[1-a][1+\frac{1-\alpha}{\alpha}a]\right\}^{-1/2-i\nu}\ ,
\end{aligned}$$

where we have made use of the symmetry between α and $1-\alpha$. The integrals above can readily be expressed in terms of Hypergeometric functions:

$$\begin{aligned}
&= 4i\ \ \sin[(-1/2-i\nu)\pi]\ [\alpha(1-\alpha)]^{-1/2-i\nu} \\
&\quad\times\Gamma^2(1/2-i\nu)\ \Gamma^2(1/2+i\nu) \\
&\quad\times{}_2F_1\left(1/2+i\nu, 1/2+i\nu; 1; \frac{\alpha}{\alpha-1}\right) \\
&\quad\times{}_2F_1\left(1/2+i\nu, 1/2+i\nu; 1; \frac{\alpha-1}{\alpha}\right)\ , \\
&= 4\imath\ \sin[(-1/2-i\nu)\pi]\ \ \frac{\pi^2}{\sin^2[(1/2-i\nu)\pi]} \\
&\quad\times{}_2F_1\,(1/2-i\nu, 1/2+i\nu; 1; \alpha) \\
&\quad\times{}_2F_1\,(1/2-i\nu, 1/2+i\nu; 1; 1-\alpha)\ . \qquad (3.11)
\end{aligned}$$

Putting the results in (3.9) and (3.11) back into (3.8) we get:

$$\begin{aligned}
&2\ e^2 g^2\ \left(\frac{2}{q}\right)^{1-2i\nu}\ \Gamma^2(1/2-i\nu) \\
&\quad\times \int_0^1 d\alpha\ [1-2(1-\alpha)+2(1-\alpha)^2] \\
&\quad\times{}_2F_1\,(1/2-i\nu, 1/2+i\nu; 1; 1-\alpha) \\
&\quad\times{}_2F_1\,(1/2-i\nu, 1/2+i\nu; 1; \alpha)\ . \qquad (3.12)
\end{aligned}$$

For the flip amplitude we have to proceed in a very similar way and find:

$$\begin{aligned}
&-\,2\ e^2 g^2\ \left(\frac{2}{q}\right)^{1-2i\nu}\ \Gamma^2(1/2-i\nu)\ (1/4+\nu^2) \\
&\quad\times \int_0^1 d\alpha\ \alpha(1-\alpha) \\
&\quad\times{}_2F_1\,(1/2-i\nu, 1/2+i\nu; 2; 1-\alpha) \\
&\quad\times{}_2F_1\,(1/2-i\nu, 1/2+i\nu; 2; \alpha)\ , \qquad (3.13)
\end{aligned}$$

which is the same for the $(+,-)$- or the $(-,+)$-amplitude.

The further calculation is rather tedious but straight forward. The main point is to expand the first Hypergeometric function, perform the α-integration and then reduce the remaining series. The final result takes on the following form:

$$e^2 g^2\ \left(\frac{2}{q}\right)^{1-2i\nu}\ \frac{\Gamma(1/2-i\nu)}{\Gamma(1/2+i\nu)}\ \frac{\pi^2}{4}\ \frac{11/4+3\nu^2}{1+\nu^2}\ \frac{\tanh(\pi\nu)}{\pi\nu} \qquad (3.14)$$

for the non-flip amplitude and

$$e^2 g^2\ \left(\frac{2}{q}\right)^{1-2i\nu}\ \frac{\Gamma(1/2-i\nu)}{\Gamma(1/2+i\nu)}\ \frac{\pi^2}{4}\ \frac{1/4+\nu^2}{1+\nu^2}\ \frac{\tanh(\pi\nu)}{\pi\nu} \qquad (3.15)$$

for the flip amplitude.

We are left with the calculation of the other end of the BFKL-Pomeron, i.e. the coupling to a quark(gluon) in the proton. We follow the consideration in [7], where it was noted that a simple projection of the eigenfunction (3.1) to a single quark line would give zero. In reality the quark is accompanied by a bunch of particles with opposite colour far away in impact-parameter space. Let us place the quark in impact-parameter space at $\mathbf{r}'_1$ and the opposite colour charge at $\mathbf{r}'_2$. In the limit $|\mathbf{r}'_2| \gg |\mathbf{r}'_1|$ the eigenfunction (3.1) reduces to:

$$E^\nu(\rho'_{10}, \rho'_2 \to \infty) \sim \left|\frac{1}{\rho'_{10}}\right|^{1-2i\nu}\ . \qquad (3.16)$$

The momentum coming from the photon is transferred to the quark, but momentum conservation is imposed later on by doing the final ρ_0-integration. At the moment we give the quark a general momentum kick $\mathbf{q}'$, which leads to a phase factor $e^{i\mathbf{q}'\cdot\mathbf{r}_1'}$. The integration over $\mathbf{r}_2'$ is factored off and only the $\mathbf{r}_1'$-integration remains:

$$\begin{aligned} &2g^2 \int \frac{d^2\mathbf{r}_1'}{(2\pi)^2} \left|\frac{1}{\mathbf{r}_1' - \rho_0}\right|^{1-2i\nu} e^{-i\mathbf{q}'\cdot\mathbf{r}_1'} \\ &= g^2 \left(\frac{2}{q}\right)^{1+2i\nu} \frac{\Gamma(1/2+i\nu)}{\Gamma(1/2-i\nu)} \frac{1}{2\pi} e^{-i\mathbf{q}'\cdot\rho_0} \ . \end{aligned} \tag{3.17}$$

A factor 2 was added to account for the coupling of both gluons.

We have to recall that we dropped the phase factor $e^{i\mathbf{q}\cdot\rho_0}$ in (3.4). In the full amplitude the two phase factors have to be drawn together and the integration over ρ_0 forces $\mathbf{q}$ and $\mathbf{q}'$ to be equal:

$$\int d^2\rho_0 e^{i(\mathbf{q}-\mathbf{q}')\cdot\rho_0} = (2\pi)^2\, \delta^2(\mathbf{q}-\mathbf{q}') \ . \tag{3.18}$$

For us it is important to note the extra factor $(2\pi)^2$ which is generated in the above equation.

4 The complete cross section

In order to write down the cross section we have to find the complete expression for the amplitude. Cross section and amplitude are linked through the relation ($|t| = \mathbf{q}^2$):

$$\begin{aligned} &\frac{d\sigma}{dt}(\gamma q \to \gamma q) \\ &= \frac{1}{2}\, \frac{|A_{(+,+)}|^2 + |A_{(-,-)}|^2 + |A_{(-,+)}|^2 + |A_{(+,-)}|^2}{16\pi s^2} \ , \\ &= \frac{|A_{(+,+)}|^2 + |A_{(+,-)}|^2}{16\pi s^2} \ . \end{aligned} \tag{4.1}$$

All missing factors not yet included are extracted from the 'Born'-diagram: 6/9 for the light flavour charges, 4/6 for the colour (in the case of γq-scattering), 4 from the coupling to the lower line, $s/4$ from the Sudakov decomposition, $1/2(2\pi)^3$ for the onshell quarks (1/2 because we need the imaginary part and not the discontinuity) and $1/(2\pi)^8$ from the phase space integral. All factors related to Fourier transformations have been taken care of, only the factor $(2\pi)^2$ from (3.18) needs to be included. Compiling all these factors and adding them together with expression (3.14),(3.15) and (3.17) we finally obtain for the amplitudes:

$$\begin{aligned} A_{(+,+)} &= i\, \frac{6}{9}\, \alpha_{em}\, \alpha_s^2\, \frac{4\pi}{3}\, \frac{s}{|t|} \int d\nu\, \frac{\nu^2}{(1/4+\nu^2)^2} \\ &\times \frac{11/4 + 3\,\nu^2}{1+\nu^2}\, \frac{\tanh(\pi\nu)}{\pi\nu} \left(\frac{s}{|t|}\right)^{\omega(\nu)} , \end{aligned} \tag{4.2}$$

and

$$\begin{aligned} A_{(+,-)} &= i\, \frac{6}{9}\, \alpha_{em}\, \alpha_s^2\, \frac{4\pi}{3}\, \frac{s}{|t|} \int d\nu\, \frac{\nu^2}{(1/4+\nu^2)^2} \\ &\times \frac{1/4 + \nu^2}{1+\nu^2}\, \frac{\tanh(\pi\nu)}{\pi\nu} \left(\frac{s}{|t|}\right)^{\omega(\nu)} , \end{aligned} \tag{4.3}$$

where

$$\omega(\nu) = \frac{3\alpha_s}{\pi} \left[2\,\Psi(1) - \Psi(1/2+i\nu) - \Psi(1/2-i\nu)\right] \ . \tag{4.4}$$

One can derive the contribution for γg-scattering simply by multiplying the previous expressions by a factor 9/4 (due to colour).

There is a striking similarity between the results here and those found for the forward jet cross section with azimuthal dependence [14]. The factor $\frac{11/4+3\,\nu^2}{1+\nu^2}$ for the non-flip contribution is identical to the corresponding integrated contribution to the forward jet cross section, whereas $\frac{1/4+\nu^2}{1+\nu^2}$, the flip result, is found for the azimuth dependent part.

The saddle point approximation for the amplitudes (4.2) and (4.3) yields:

$$A_{(+,+)} = 11\, i\, \frac{6}{9}\, \alpha_{em}\, \alpha_s^2\, \frac{s}{|t|}\, \frac{8}{3} \left(\frac{\pi}{7\zeta(3)\,\eta}\right)^{3/2} e^{\eta \ln(4)} \ , \tag{4.5}$$

and

$$A_{(+,-)} = i\, \frac{6}{9}\, \alpha_{em}\, \alpha_s^2\, \frac{s}{|t|}\, \frac{8}{3} \left(\frac{\pi}{7\zeta(3)\,\eta}\right)^{3/2} e^{\eta \ln(4)} \ , \tag{4.6}$$

where η is defined as:

$$\eta = \frac{6\alpha_s}{\pi} \ln\left(\frac{s}{|t|}\right) \ . \tag{4.7}$$

In this approximation one nicely sees the dominance of the non-flip versus the flip amplitude given by the factor of 11 in (4.5). On the scale we perform our numerical analysis the difference between the exact result and the saddle point approximation is marginally less than a factor of 2.

5 Numerical results

Since we are mainly interested in a rough estimate for the cross section, we will concentrate on elastic photon-quark scattering. In order to get an impression for the size of the cross section we compare with a different process, namely diffractive J/Ψ-photoproduction. In addition, we use the Vector meson Dominance Model (VDM) to gain an estimate of the quasi elastic γq scattering at low $|t|$, and we perform a comparison between the full BFKL solution and the leading order two-gluon exchange.

The formulae for the J/Ψ-cross section are taken from [2] and [3]. The leading order two gluon exchange can be more easily calculated from the momentum representation of the photon wave function (2.8). The amplitudes in this case are given by:

$$\begin{aligned} A_{(+,+)} &= i\, \frac{s}{|t|}\, \alpha_{em}\alpha_s^2\, \frac{2^6}{3^3} \left(\frac{\pi^2}{3} + 1\right) , \\ A_{(+,-)} &= i\, \frac{s}{|t|}\, \alpha_{em}\alpha_s^2\, \frac{2^5}{3^2} \ . \end{aligned} \tag{5.1}$$

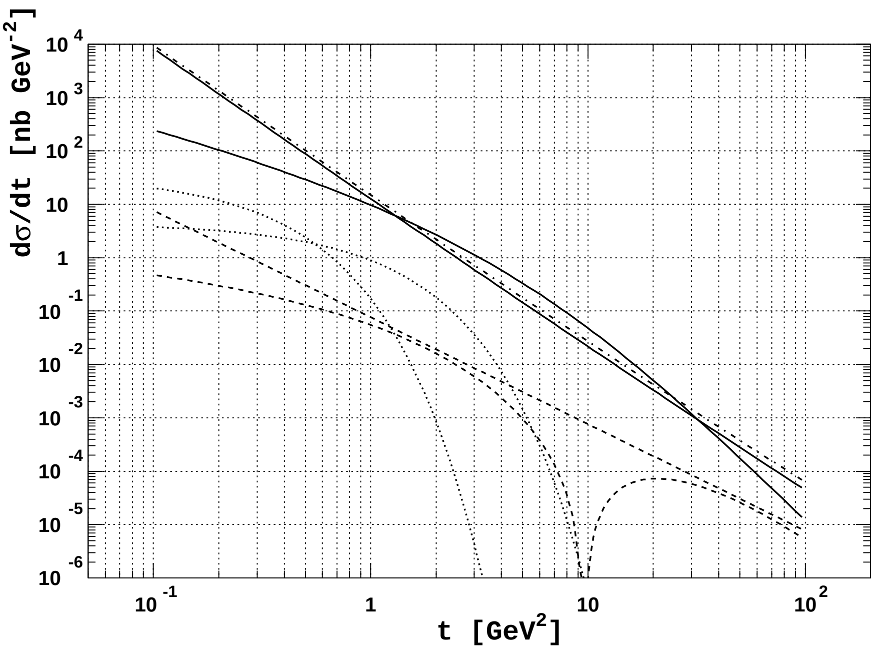

Fig. 2. The cross section for diffractive production of photons and J/Ψ's at $W = 200\ GeV$. The solid line denotes the BFKL-solution (straight line for photons), the dashed line the leading order two-gluon exchange (straight line again for photons), and the dotted line shows the VDM-estimate (the upper line is for J/Ψ). In addition, the saddle point approximation is given as dash-dotted line

A VDM-estimate can be achieved by employing the optical theorem which relates the elastic with the total γp-cross section. The total γp-cross section at the energy of $W = 200\ GeV$ is approximately 150 μb. This leads together with the t-slope of 10 GeV^{-2} to an integrated elastic γp-cross section of 115 nb. From the diffractive production of ρ^0's one knows that the ratio of proton dissociation versus elastical proton scattering is 1/2 [15]. Following the additive quark model we divide the elastic cross section by another factor of 9 (in total 18) which then gives an estimate for the γq-scattering of 6.4 nb at $W = 200 GeV$. The diffractive slope in $|t|$ is according to the ρ^0-data 5.3 GeV^{-2} for events with proton dissociation. Altogether one finds:

$$\frac{d\sigma}{dt}(\gamma) \approx 34.1\ nb\ \exp(-5.3\ |t|\ GeV^{-2})\ . \qquad (5.2)$$

Taking the measured cross section for $J/\Psi p$ at the same energy and dividing it by 9, we find a somewhat lower value. The main difference is the much smaller slope of 1.6 GeV^{-2} which is due to the large mass:

$$\frac{d\sigma}{dt}(J/\Psi) \approx 4.4\ nb\ \exp(-1.6\ |t|\ GeV^{-2}) \qquad (5.3)$$

for $W = 200 GeV$. In Fig. 2 we show all discussed options in one plot. For the perturbative result we have assumed a constant value for the strong coupling of $\alpha_s = 0.2$ and $W = 200\ GeV$. The two solid lines are related to the full BFKL-solution, the straight line denotes the production of a photon and the curved line the production of J/Ψ. In addition to the straight solid line we have plotted the saddle point solution (4.5) and (4.6) for γq-scattering which lies almost on top of the solid line. The dashed lines show in a similar fashion the cross sections based on the leading order two-gluon exchange. Again the straight line denotes the production of a photon and the curved line the production of J/Ψ. The strongly curved, dotted lines represent the VDM-estimates. The line related to the production of J/Ψ lies above the line for the production of a photon due to the smaller t-slope.

The VDM-result for γq-scattering seems to contradict the perturbative result. Moreover, a matching between the low $|t|$ nonperturbative and high $|t|$ perturbative regime seems to be difficult. For J/Ψ, on the other hand, a matching seems feasible. The enhancement due to BFKL is extremely large, in both cases it is a factor of about 100 at $|t| = 3\ GeV^2$ and somewhat smaller (a factor 10) at $|t| = 100\ GeV^2$.

The fact that the perturbative BFKL-result for γq-scattering overshoots the VDM-result so massively makes it hard to believe that the perturbative prediction is close to the true value. There are two main reasons that should be mentioned. First, the BFKL-solution is implemented at Leading Order. NLO-corrections are known to reduce the cross section substantially. Second, although we consider large-$|t|$, the internal integration over the transverse momenta in the virtual loops is performed without any infrared cutoff. The result is still finite, but dominant contributions might come from the infrared region and thus is influenced by confinement. This point needs further investigation by imposing an infrared cutoff on the separation of the quark-antiquark pair. The consequence of all conceivable corrections might be a reduction of the cross section by 1 or 2 orders of magnitude.

6 Conclusions

We have calculated the cross section for γq-elastic scattering in the Regge limit ($W^2 \gg |t| \gg \Lambda^2_{QCD}$). The BFKL-solution for nonzero momentum transfer was used leading to a strong enhancement of the cross section. The comparison with a VDM-calculation indicates, however, that the perturbative prediction might be much to high, and data from HERA would be very valuable in assessing the validity of the present pQCD result. We also found that the cross section for elastic γq-scattering is similar in magnitude to the cross section for $\gamma q \to J/\Psi q$.

Some effort was put into the detailed presentation of the method employed for performing the convolution of the photon wave function with the conformal eigenfunction. The integrals we were faced with are typical for two-dimensional conformal field theories. They seem to be solved most efficiently when complex variables are introduced and the integration is factorized in complex times complex-conjugate contributions. A similar technique was used in [16]. We have pointed out that special attention has to be given to the problem of large infrared contributions. One way of tackling this problem is to keep the virtuality Q^2 of the initial photon high enough [17]. It will be interesting to see how much the $\gamma^* q$-cross section will decrease when the photon virtuality is increased from 0 to 1 GeV^2.

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